PHYS-434 Physics of photonic semiconductor devices

Series 7 - Basic notions on microcavity

Exercise I: Light matter interaction in microcavities

1. Solutions to the system $\det(H-\lambda I)=0$ are such that:

$$\begin{vmatrix} E_X-\lambda & \hbar\Omega/2 \\ \hbar\Omega/2 & E_c-\lambda \end{vmatrix} = 0 \quad \Rightarrow (E_X-\lambda)(E_c-\lambda)-\left(\frac{\hbar\Omega}{2}\right)^2=0$$

$$\Rightarrow E_X E_c + \lambda^2 - \lambda(E_X+E_c) - \left(\frac{\hbar\Omega}{2}\right)^2 = 0$$

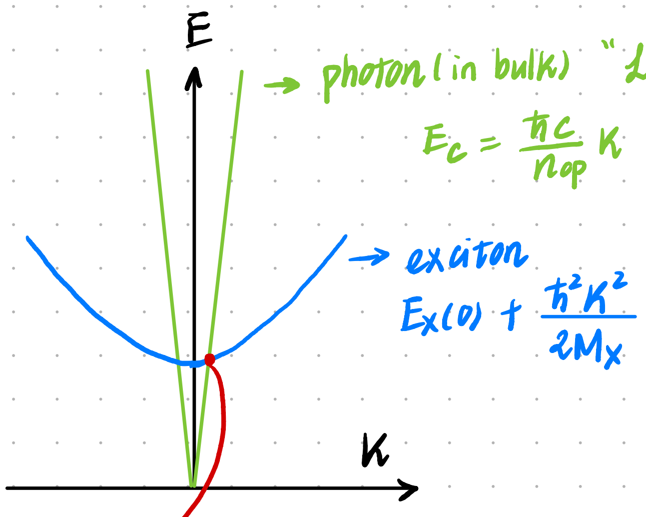
As a result: $E_\pm = \frac{1}{2}(E_X+E_c) \pm \frac{1}{2}\left[(E_X+E_c)^2 - 4\left(E_X E_c - \left(\frac{\hbar\Omega}{2}\right)^2\right)\right]^{\frac{1}{2}}$

which leads to: $E_\pm = \frac{1}{2}(E_X+E_c) \pm \frac{1}{2}\left[(E_X-E_c)^2+(\hbar\Omega)^2\right]^{\frac{1}{2}}$

---

2. cf attached figure on the next page.

3.



Optical transitions imply energy & momentum conservation

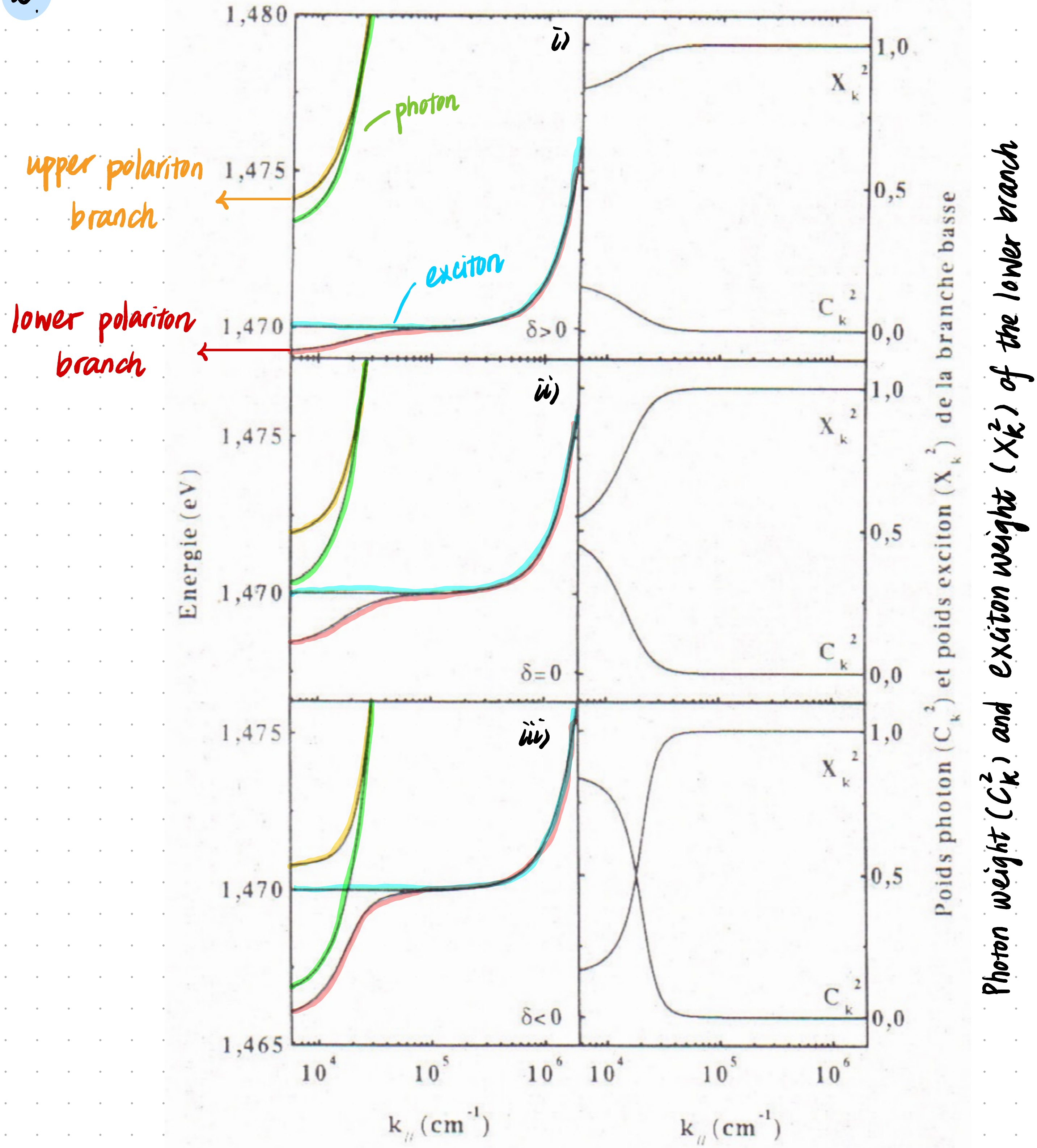
$\Rightarrow$ radiative recombinations of excitons only allowed for excitons of K such that $E_X(K)$ lies within the light cone!

$\Rightarrow$ max: $\frac{\hbar c}{n_{op}} k_{rad} = E_X(0) + \underbrace{\frac{\hbar^2}{2M_X} k_{rad}^2}$ → can be neglected as excitons exhibit a relatively heavy mass

$\Rightarrow \quad k_{rad} \approx \frac{n_{op} \cdot E_X(0)}{\hbar c}$

and the energetic range of these radiative states is about $\frac{\hbar^2 k_{rad}^2}{2M_X}$

2.



- Left: dispersion relation (energy as a function of K) of the polaritons for three cases: i) $E_c > E_K$, ii) $E_c = E_K$, iii) $E_c < E_K$.
- Right: photon weight (C_K^2) and exciton weight (X_K^2) of the lower polariton branch as a function of K.

3.

$$k_{rad} \approx \frac{n_{op} \cdot E_X(0)}{\hbar c}, \quad \Delta E \approx \frac{\hbar^2 k_{rad}^2}{2 M_X}$$

GaAs case

$$k_{rad} \approx \frac{3.57 \times 1.47 \times 1.602 \times 10^{-19}}{1.054 \times 10^{-34} \times 3 \times 10^{8}} \approx 2.7 \times 10^{7}\, m^{-1}$$

$$M_X = m_e^* + m_{hh}^* = 0.567\, m_0$$

$$\Delta E \approx \frac{(1.054 \times 10^{-34} \times 2.7 \times 10^{7})^2}{2 \times 0.567 \times 9.1 \times 10^{-31}} \approx 50\, \mu eV$$

GaN case

$$k_{rad} \approx \frac{2.5 \times 3.00 \times 1.602 \times 10^{-19}}{1.054 \times 10^{-34} \times 3 \times 10^{8}} \approx 3.8 \times 10^{7}\, m^{-1} \qquad M_X \approx 1.3\, m_0$$

$$\Delta E \approx 40\, \mu eV$$

4. Reminder (series 6): $E_c(k_\parallel) = \underbrace{E_c(0)}_{= \frac{\hbar c p \pi}{n_{cav} L_{cav}}} + \frac{\hbar^2 k_\parallel^2}{2 \underbrace{m_{ph}}_{= \frac{\hbar p \pi n_{cav}}{c L_{cav}}}}$

For zero detuning: $E_c(0) = E_X(0)$

$$\Rightarrow E_X(k_\parallel) \approx \cancel{E_X(0)} < E_c(k_\parallel) - \Delta E/2 = \cancel{E_c(0)} + \frac{\hbar^2 k_\parallel^2}{2 m_{ph}} - \frac{\Delta E}{2}$$

Kinetic energy term neglected as M_X relatively heavy

$$\Rightarrow k_\parallel < \left(\frac{m_{ph} \Delta E}{\hbar^2}\right)^{\frac{1}{2}}$$

With $m_{ph} = \frac{\hbar p \pi}{L_{cav}} \frac{n_{cav}}{c} = \frac{\hbar p \pi c}{n_{cav} L_{cav}} \cdot \frac{n_{cav}^2}{c^2} = E_c(0) \cdot \frac{n_{cav}^2}{c^2} = E_X(0) \cdot \frac{n_{cav}^2}{c^2}$

$$\Rightarrow k_c = \left(\frac{m_{ph} \cdot \Delta E}{\hbar^2}\right)^{\frac{1}{2}} = \left(\frac{\Delta E \cdot n_{cav}^2 \cdot E_X(0)}{\hbar^2 \cdot c^2}\right)^{\frac{1}{2}}$$

In addition, $k_c = \frac{E_x \cdot n_{op}}{\hbar c} \cdot \sin\theta_c^i$ Snell: $n_{op} \cdot \sin\theta_c^i = 1 \cdot \sin\theta_c$

and $E_x \sim E_x(0)$

$$\Rightarrow \quad k_c \approx \frac{E_x(0)}{\hbar c} \sin\theta_c \Rightarrow \theta_c = \sin^{-1}\left(\frac{\hbar c k_c}{E_x(0)}\right)$$ ... corresponding external angle

---

5. The time required for a photon to perform a single cavity round-trip is:

$$t_{r.t.} = \frac{2 n_{cav} L_{eff}}{c} = \frac{2 n_{cav} \cdot (L_{cav} + 2 L_{DBR})}{c}$$ (2 mirrors)

(L_{eff} are the effective length of the Fabry-Perot cavity.)

$$n_{cav} = n_{GaAs}, \quad L_{cav} = \frac{3}{2}\frac{\lambda}{n_{cav}} = \frac{3}{2}\frac{\lambda}{n_{GaAs}} \Rightarrow n_{cav} \cdot L_{cav} = \frac{3}{2}\lambda$$

$$n_{cav} \cdot L_{DBR} = \cancel{n_{cav}} \cdot \frac{n_1 n_2}{\cancel{n_{cav}}(n_2 - n_1)} \frac{\lambda}{4} = \frac{n_1 n_2}{4(n_2 - n_1)} \lambda$$

$$\Rightarrow t_{r.t.} = \frac{2}{c}\left(\frac{3}{2}\lambda + \frac{n_1 n_2}{2(n_2 - n_1)}\lambda\right) \Rightarrow t_{r.t.} = \left(3 + \frac{n_1 n_2}{n_2 - n_1}\right)\frac{\lambda}{c}$$

(for GaAs/AlAs DBRs, $n_1 = n_{AlAs}$, $n_2 = n_{GaAs}$)

By definition: $\frac{dN}{dt} = -\frac{N}{\tau_{cav}} \Rightarrow N(t) = N_0 e^{-\frac{t}{\tau_{cav}}}$

after 1 round-trip: $N(t_{r.t.}) = N_0 e^{-\frac{t_{r.t}}{\tau_{cav}}}$

$\Rightarrow$ Losses over a single cavity round-trip are thus expressed as:

$$\alpha_{losses} = \frac{N(t=0) - N(t=t_{r.t.})}{N(t=0)} = \frac{N_0 - N_0 e^{-\frac{t_{r.t}}{\tau_{cav}}}}{N_0} = 1 - \exp\left(-\frac{t_{r.t.}}{\tau_{cav}}\right) \approx \frac{t_{r.t}}{\tau_{cav}}$$

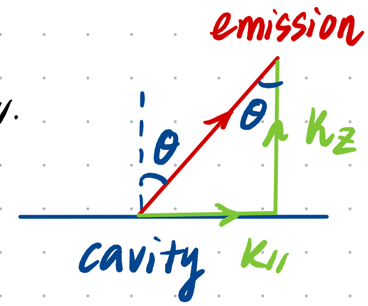
$$\Rightarrow \quad \alpha_{losses} \approx \left(3 + \frac{n_1 n_2}{n_2 - n_1}\right)\frac{\lambda}{c \cdot \tau_{cav}}$$

$$= \left(3 + \frac{3.57 \cdot 2.95}{3.57 - 2.95}\right)\frac{850 \times 10^{-9}}{3 \times 10^{8} \times 6 \times 10^{-12}} \approx 0.9\%$$

Exercise II: Properties of microcavity emission

The Fabry-Perot cavity photon dispersion is given by:

$$E_c(k_{//}) = E_c(0) + \frac{\hbar^2 k_{//}^2}{2 m_{ph}} \quad \text{where} \quad m_{ph} = \frac{p\hbar\pi n_{cav}}{c L_{cav}}$$



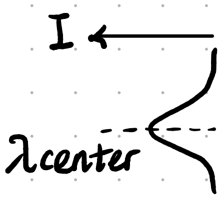
So, for a given cavity mode, the dispersion is quadratic in $k_{//}$.

Now, $k_{//}$ is proportional to $\sin\theta$ (θ = emission angle):

Hence $$E_c(\theta) = E_c(0) + \frac{\omega^2\hbar^2\sin^2\theta}{2c^2 m_{ph}}$$

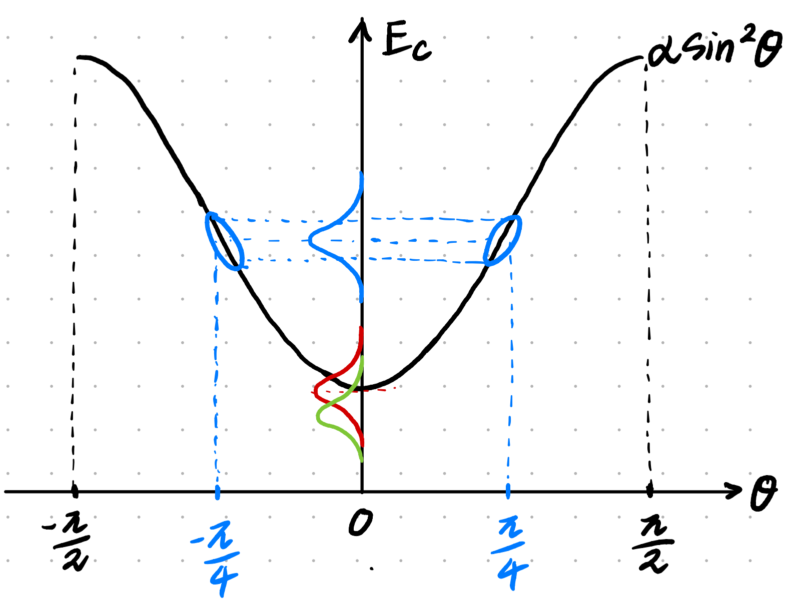
This then immediately answers Q-2), explaining the $\sin^2\theta$ shape of the angular dependence of the output mode energy.

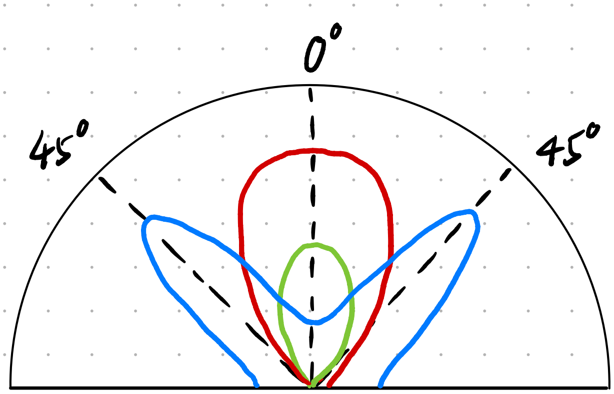
To answer Q-1), we need to think about how the excitation source will change the emission angle (N.B., it is the input photon energies being varied in this question, NOT the detected output photons as in Q-2))



= Energy distribution of polychromatic source

$\lambda_{center} = 460\,nm$
$\lambda_{center} = 450\,nm$
$\lambda_{center} = 430\,nm$





The above diagram explains the behaviour.

- For this particular microcavity, $E_c(0)$ occurs at around $\lambda = 450\,nm$.
- Hence when the cavity is exposed to polychromatic light with $\lambda_{center} \sim 450\,nm$, the excited cavity mode has a small $k_{\parallel}$ and hence emits around $\theta = 0°$.
- Using a source with $\lambda_{center} \sim 460\,nm$ produces a similar result, but now much of the input radiation is below the minimum cavity mode energy and therefore does not contribute to the output emission.

  This leads to the same emission pattern but with lower overall intensity.
- If we use a source with $\lambda_{center} \sim 430\,nm$, the higher input energy means a greater $k_{\parallel}$ component is required, and hence output emission mostly occurs at considerable values of θ — around 45° in this case.